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Separation Science and Technology
Publication details, including instructions for authors and subscription information:
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To cite this Article Abbasov, Teymuraz and Ceylan, Kadim(1998) 'Estimation of Optimum Fluid Velocity in High Gradient Magnetic Filtration', Separation Science and Technology, 33: 7, 975 — 989
To link to this Article: DOI: 10.1080/01496399808545002
URL: http://dx.doi.org/10.1080/01496399808545002

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SEPARATION SCIENCE AND TECHNOLOGY, 33(7), pp. 975–989, 1998

Estimation of Optimum Fluid Velocity in High Gradient Magnetic Filtration

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ABSTRACT

A high gradient magnetic cleaning process is favored by several branches of industry for the high purification of various technological fluids involving magnetic (ferro, para, or diamagnetic) particles. This paper presents a theoretical study about the effects of the parameters of a magnetic filtration system on the fluid velocity to be chosen. A model is presented to estimate the optimum filtration velocity needed to satisfy a predefined filter performance. The model is applicable for magnetic filter systems with packing fractions of up to 0.62 and for fluids containing magnetic particles. It is essentially based on the balance of moments acting on particles captured and accumulated in the magnetic filter. The boundary layer approach is used in the development of the model. The results indicate that the optimum filtration velocity depends on the properties of the fluid and on the parameters of the filter system. Model predictions and experimental data given in the literature are in a good agreement.

Key Words. Magnetic; Filter; Filtration velocity

INTRODUCTION

The magnetic separation technique is used for the enrichment of ores or for cleaning various technological materials in several branches of industry

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(1, 2). The high gradient magnetic filtration (HGMF) process has been favored in recent years to remove micron-sized para and ferromagnetic particles from fluid suspensions (3, 4). A great deal of effort has been devoted to improve the technology, and studies on the subject are currently continuing (5, 6).

A high gradient magnetic filter is made up of a canister containing small magnetic elements, such as ferromagnetic fibers, rods, spheres, stainless steel wool, etc., placed into a uniform magnetic field of 0.1–1.5 T (7, 8). In the presence of an applied magnetic field, these elements create a high gradient magnetic fields at their surroundings. HGMF is more effective if the ferromagnetic elements in the filter are in contact (8, 9). An active area is created around these contacting points in which the particles are captured and collected. One of the essential properties of magnetic filters is their strong resistance to thermal and chemical effects. This technique can be used under conditions of high filtration velocities (up to 0.1 m/s) and high temperatures (up to 600 K). They also have the advantage over classical filters due to their easier regeneration. The chemical and physical properties of the fluid, the geometrical and magnetic properties of the filter elements, and the filtration velocity are important parameters for the effectiveness of the filter.

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Although some specific models are given in the literature for the mechanism of the filtration process, there is no general model which includes the relations among all the parameters affecting the mechanism (2, 3). The existing filtration models generally focus on the capture of heavier particles or on the drag of captured particles by the fluid flow. Theoretical studies on the mechanism of magnetic particle capture in porous media are mainly based on an analysis of force and momentum balance for particles captured in the filter. This model is known as the "particle trajectories model," and it was essentially developed for magnetic separators (3, 4). Although the magnetic separation and magnetic filtration processes are basically similar, the application conditions and the particle capture mechanism are relatively different. A magnetic separator has a porosity of 0.90–0.95 and is capable of separating particles of >10 μm. A magnetic filter has a porosity of 0.38–0.48 and is capable of capturing particles of <10 μm. In addition, the working concentration range of HGMF is very low, about 10^{-3}–10^{-5} mg/kg (9).

A measure of the effectiveness of a filtration process is given by filter performance, defined as $\Delta C/C_0$, where C_0 is the total concentration (g/kg) of particles in the fluid before filtration and ΔC is the decrease in particle concentration after filtration. In the case of magnetic filtration, the concentration of magnetic particles is important in order to determine the filtering efficiency (the recovery). A definition of the magnetic particle fraction may be introduced as $\Phi = C_0'/C_0$, where C_0' is the total concentration (g/kg) of magnetic particles. For suspensions of pure magnetic particles, $\Phi = 1$ However, for actual technological fluids to be cleaned by magnetic filtration, Φ

ranges from 0.6 to 0.9 (8, 9). The experimental data given in the literature indicate that fluid flow velocity is one of the most important parameters which affects the performance of a magnetic filter. The velocity must be lower than a limiting value to achieve a desired filtration performance. The highest fluid velocity (bulk flow velocity) satisfying a predefined filter performance is called the "optimum filtration velocity," and it is generally determined by experimental methods (8, 9). At higher fluid velocities the particles accumulated within a filter will be dragged out due to hydrodynamic effects, and performance will decrease although the filter is not saturated. The velocity which begins dragging of the accumulated particles in the filter is called the "critical velocity." It appeared to us to be useful to develop some new models for estimating the optimum filtration velocity in terms of the parameters of the filter system.

In this paper a model is developed for estimating the optimum filtration velocity for a HGMF system. The model is applicable for magnetic filter systems with packing fractions of up to 0.62 and for fluids containing magnetic particles. The model is still valid in the case of diamagnetic particles; however, construction of the filter system may require some modifications. The model is in fact an equilibrium model, and it is essentially based on the force and moment balances for the particles captured and accumulated in the vicinity of the touching points of the filter elements. The boundary layer approach is used to describe the accumulation mechanism of the particles and to estimate the optimum filtration velocity. The model predictions are verified with experimental data given in the literature.

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A THEORETICAL MODEL FOR CALCULATION OF OPTIMUM FILTRATION VELOCITY

In order to formulate the theoretical approach presented here, the following simplifying assumptions were made.

1. A magnetic filter consists of contacting magnetized ferromagnetic spherical elements with identical capacities for particle capture.
2. The particles in the suspension are spherical and of equal size.
3. The particles are captured around the touching points of the filter elements and accumulated in layers. The surfaces of these layers are elementary flat plates.
4. The sum of the surfaces of all the elementary plates are named the active accumulation surfaces, and the sum of the lengths of these plates in the flow direction is named the active filter length, L_a.
5. The flow regime on the outer most layer is characterized by the formation of a boundary layer. Fluid flow between the layers is neglected.

6. The forces which affect particle capture and accumulation are the magnetic force (F_m), the drag force (F_D), and the gravitational force (F_G). All other forces (e.g., adhesion, electrical, etc.) and diffusion are neglected.
7. The sum of the moments exerted on each of particle at rest is equal to zero.

Accumulation of captured particles around the contacting points of the filter elements, in the active area, is schematically demonstrated in Fig. 1. The particles accumulated at the bottom are stable. However, the particles on the outermost layer may be unstable since they are subjected to a direct drag force, and therefore they may be dragged due to the hydrodynamic effects of the interstitial fluid velocity.

The average fluid velocity within a filter (V_i) is dependent on both the bulk flow velocity (V_f, the filtration velocity) and on the filter porosity (ϵ) as follows:

$$V_i = V_f/\epsilon$$

The porosity of a filter decreases as particles accumulate in the filter (see Fig. 1); however, changes in magnetic filters can be neglected. Data in the literature indicate that the experimentally determined optimum filtration velocity, V_f, for a number of magnetic filters is around 0.06–0.08 m/s (9). Based on the average interstitial velocity and on the diameter of the filter elements, one may calculate that the Reynolds (Re) number in these filters is around 400–500 and that the flow is highly turbulent. Since particle capture is very unlikely under these conditions, the velocity profile in the filter must be taken into consideration to account for the accumulation phenomenon because fluid

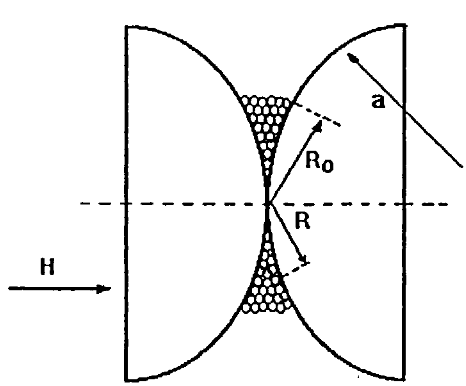

FIG. 1 Schematically representation of particle accumulation in the active area.

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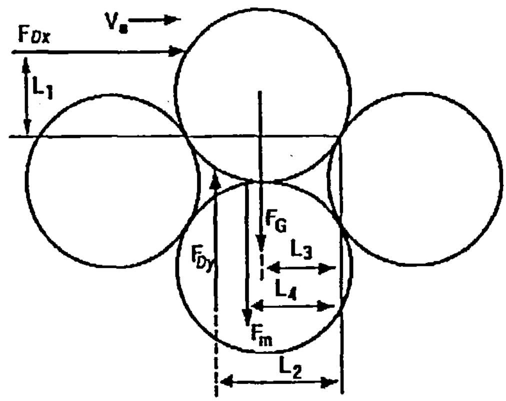

FIG. 2 Schematically representation of the forces acting on a particle accumulated on the utmost layer.

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motion is greatly affected by the solid surface and a velocity distribution occurs within the filter. The velocity of fluid at the fluid–solid interface is zero, and it increases up to a maximum value at the center of the spaces between the spheres (the cross-sectional area available for flow) in the filter. The flow is laminar up to a certain distance from the solid surface, but a turbulent boundary layer then follows. It is assumed that particle accumulation occurs mainly in this laminar sublayer. At a certain distance from the contact points of the spheres (R_0 in Fig. 1), the critical fluid velocity is attained, and thereafter the hydrodynamic forces (the drag forces) dominate the other forces. The filter behaves as if it is saturated under these conditions, and particle accumulation no longer occurs. The distance at which the critical velocity is attained essentially depends on the filtration velocity for a certain filter system.

The above discussions imply that the flow regime and the forces acting on the particles are the main parameters which affect particle accumulation. These forces are demonstrated for a particle at the outermost layer in Fig. 2. A relation among these forces must be developed in order to clarify the mechanism of the accumulation and to estimate the optimum filtration velocity. The estimation of these forces is summarized below.

The Magnetic Force

The magnetic force on a ferromagnetic particle in the vicinity of the contacting points of the filter elements is given by Sandulyak (9) as:

$$F_m = \frac{\pi\delta^3}{6} \frac{\mu_0(\chi_p - \chi_f)\mu^2(\mu - 1)H^2(R/a)}{a\{1 + 0.5(\mu - 1)(R/a)^2\}^3} \quad (1)$$

where $\mu_0 = 4\pi \times 10^{-7}$ (H/m), the free space magnetic permeability, χ_p = particle magnetic susceptibility, χ_f = fluid magnetic susceptibility, μ = magnetic permeability of the filter elements, H = external magnetic field (A/m), a = radius of the filter elements (m), δ = effective particle size (m), and R = radial coordinate (m) from the contacting point as indicated in Fig. 1. Since $\chi_p \gg \chi_f$ in general, a definition of effective magnetic susceptibility, χ, as $\chi = \chi_p - \chi_f$ is useful for the sake of simplicity of the calculations. It is clear in this definition that χ can be understood to approximate χ_p for most practical applications.

The Drag Force

The drag force affecting a particle may be calculated by following two different procedures.

In the first, the drag force is calculated in general form (10):

$$F_{Dx} = \frac{\rho_f\pi\delta^2}{4} V_0^2\lambda_x M \quad (2)$$

$$F_{Dy} = \frac{\rho_f\pi\delta^2}{4} V_0^2\lambda_y M \quad (3)$$

where ρ_f is fluid density (kg/m^3), λ_x and λ_y are the drag and lift components of the drag force, respectively, M is a correlation coefficient regarding the fluctuations in velocity, and V_0 is the highest velocity component of the velocity gradient within the filter.

In the second, the drag force on the particle is calculated from the modified Stokes equation (11):

$$F_D = 6\pi C\tau_w \frac{\delta^2}{4} \quad (4)$$

where τ_w is the shear stress on the accumulation surface and C is a constant with a value of 1.701.

The shear stress at the surface of an accumulation layer may be calculated by means of the boundary layer theory (12):

$$\tau_w = 0.332 \frac{\rho_f V_0^2}{(\mathrm{Re}_x)^{0.5}} \quad (5)$$

where $\mathrm{Re}_x = \rho_f V_0 X/\eta$, η = viscosity of the fluid, and X = the distance from the leading edge of accumulation layer.

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The drag forces calculated in Eqs. (2), (3), and (4) are for a single particle. In fact, the downstream part of a flat plate experiences a smaller drag than the leading portion because the boundary layer is thicker toward the trailing edge (12). Therefore, defining an average drag force may be more convenient for a filter system. The average drag force may be estimated either by considering the active filter length (L_a):

$$F_D\text{av.} = \frac{1}{L_a}\int_0^{L} F_D \, dX \tag{6}$$

or it can also be calculated by considering the fluctuations in velocity of the fluid in the filter:

$$F_D\text{av.} = \frac{M}{L_a}\int_0^{L_a} F_D \, dX \tag{7}$$

The Gravitational Force

The gravitational force acting on a particle in a fluid is given by the buoyancy force:

$$F_G = \frac{\pi\delta^3}{6} g(\rho_p - \rho_f) \tag{8}$$

where ρ_p particle density (kg/m^3) and g is gravity of acceleration (m/s^2).

The Moment Balance

Referring to Fig. 2 and the forces defined in Eqs. (1), (2), and (3), the moment equation for a particle at rest on the outermost layer can be written in two forms. In the first, both the drag and lift components of the drag forces are taken into consideration in the momentum balance (10):

$$F_{Dx}L_1 + F_{Dy}L_2 - F_G L_3 - F_m L_4 = 0 \tag{9}$$

where L_1, L_2, L_3, and L_4 are the distances of the application points of the forces to the touching points of the deposited particles.

Substituting Eqs. (1), (2), (3), and (8) into Eq. 9 with the data from Gontar (10) ($\lambda_x = 0.42$, $\lambda_y = 0.1$, $L_1 = 0.74\delta/2$, $L_2 = 0.5\delta/2$, $L_3 = 0.5\delta/2$, $L_4 = 0.785\delta/2$, $M = 4$) and rearranging for V_0, we obtain

$$V_0 = 0.602 \left\{ \frac{\mu_0 \chi \mu^2 (\mu - 1) H^2 (R/a)\delta}{\rho_f a \{1 + 0.5(\mu - 1)(R/a)^2\}^3} + 0.65 \frac{g(\rho_p - \rho_f)\delta}{\rho_f} \right\}^{0.5} \tag{10}$$

In the second, only the drag component of the drag force is taken into consideration in the momentum balance (11):

$$F_m \frac{\delta}{4} + F_G \frac{\delta}{2\sqrt{2}} - F_D \frac{\delta}{2}\left(\frac{1}{\sqrt{2}} + 0.74\right) = 0 \qquad (11)$$

Equations (1), (6), and (8) are substituted into Eq. (11) and then rearranged for V_0 to give

$$V_0 = 1.422 \times 10^{-2} \left\{ \frac{\mu_0 \chi \mu^2 (\mu - 1) H^2 (R/a) \delta}{a\{1 + 0.5(\mu - 1)(R/a)^2\}^3} + 0.177 g(\rho_p - \rho_f)\delta \right\}^{2/3} \left(\frac{L}{\rho_f \eta}\right)^{1/3} \qquad (12)$$

It may be plausible to ignore the gravitational forces in Eqs. (10) and (12) since the size and mass of the captured particles are indeed very small. If the gravitational forces are neglected, Eq. (10) simplifies to

$$V_0 = 0.602 \left\{ \frac{\mu_0 \chi \mu^2 (\mu - 1) H^2 (R/a) \delta}{\rho_f a\{1 + 0.5(\mu - 1)(R/a)^2\}^3} \right\}^{0.5} \qquad (13)$$

and Eq. (12) simplifies to

$$V_0 = 1.422 \times 10^{-2} \left\{ \frac{\mu_0 \chi \mu^2 (\mu - 1) H^2 (R/a) \delta}{a\{1 + 0.5(\mu - 1)(R/a)^2\}^3} \right\}^{2/3} \left(\frac{L}{\rho_f \eta}\right)^{1/3} \qquad (14)$$

Equations (13) and (14) can both be used to estimate the highest velocity component within the filter, depending on the parameters of the filtration system. In fact, V_0 is also a function of the filtration velocity (V_f). From the point of practical applications, it is advantageous to estimate V_f instead of V_0. Based on measurements of the velocity distribution by the laser doppler method, the following relation has been suggested for the local velocity within a magnetic filter, up to Re < 300 (9):

$$V_R = K_v V_f (R/a)^2$$

where K_v is a constant and V_f is the filtration velocity. The highest fluid velocity component is observed at the center of spaces between the filter elements (at $R = a$), and it is then given as

$$V_0 = K_v V_f$$

Depending on the diameter of the filter elements and on the filtration velocity, the value of K_v lies between 10 and 20. Based on boundary layer theory, it may be estimated the flow is laminar (Re < 100) up to $R/a = 0.3$ if the

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filtration velocity is around 0.06–0.08 m/s. The experimental data in the literature also indicate that particle accumulation in the filter occurs up to $R/a = 0.3$–0.4, and filter performance is around 0.8 under these conditions (9). Therefore, the optimum filtration velocity may be estimated by assuming $K_v = 20$ and $R/a = 0.3$, and inserting these values into Eq. (13). The result gives the following equation for V_f:

$$V_f = 0.021 \left\{\frac{\mu_0 \chi \mu^2(\mu - 1)H^2\delta}{\rho_f a\{0.95 + 0.05\mu\}^3}\right\}^{0.5} \quad (15)$$

Substitution of the same values for K_v and R/a into Eq. (14) gives

$$V_f = 3.20 \times 10^{-4} \left\{\frac{\mu_0 \chi \mu^2(\mu - 1)H^2\delta}{a\{0.95 + 0.05\mu\}^3}\right\}^{2/3} \left(\frac{L}{\rho_f \eta}\right)^{1/3} \quad (16)$$

Equations (15) and (16) are very convenient for estimating optimum filtration velocities under various HGMF conditions. Equation (15) may not be considered a general formula since it does not include fluid viscosity and filter length, important parameters for a filtration system. Equation (16) is derived on the basis of the boundary layer theory and is more general since it includes all the main parameters of the system. Therefore, Eq. (16) seems to be more flexible and advantageous. However, the results indicate that the estimated velocities from Eqs. (15) and (16) are similar for cleaning aqueous suspensions in filters with a length of 0.25 m or shorter. It should also be noted that if Eq. (7) instead of Eq. (6) is used in the development of Eq. (16), the estimated optimum velocity does not change much. This result implies that Eqs. (6) and (7) are almost identical.

The preceding discussions indicate that filter performance and filtration velocity are interrelated for a certain filtration system. However, this relationship is not explicitly seen in Eq. (15) or (16). It may be useful to estimate in advance what filtration velocity should be applied to maintain a predefined filter performance. Based on geometrical relations between the accumulated particles and filter elements, the filtration velocity given above (Eq. 16), and the fraction of magnetic particles in the fluid (Φ), a general relation has been derived for the dependence of the filter performance of an unsaturated filter on the main parameters of the filtration system. The details and discussion of this derivation will be given in a subsequent paper, but the result is given here.

$$\frac{\psi}{\Phi} = 1 - \exp\left[-0.212L\left(\frac{\chi \mathbf{H}^{0.75}\delta}{\rho_f V_f^2 d^{2.7}}\right)^{0.6}\right] \quad (17)$$

where ψ is the filter performance, $\mathbf{H}$ is the dimensionless magnetic field strength, and d is the diameter (m) of the filter element. This equation can

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be safely used to estimate the filtration velocities which satisfy a predefined filter performance of a HGMF system.

RESULTS AND DISCUSSION

The equations developed above and experimental data in the literature indicate that the performance of a magnetic filter is a function of the parameters of the system and the filtration velocity. Some relations similar to Eq. (17) are also given in the literature for the velocity–performance relation. However, the dependence of performance on velocity is more sensitive in Eq. (17) than in those given in the literature. In experimental studies, the effect of each parameter, in general, was investigated one by one. However, Eqs, (15), (16), and (17) enable us to investigate the effects of each parameter on the filtration velocity or on the performance. For example, the variation of filter performance with filtration velocity, according to Eq. (17), is represented in Fig. 3 for the filtration of three different liquids in different filtration systems. The experimental data from the literature are also represented in the figure in order to allow an easy comparison with the model's predictions. As experimentally determined, the figure also indicates that there is a limiting filtration velocity (the optimum velocity) reguired to maintain a certain filter performance. It is seen from the figure that the model's predictions are in very good agreement with experimental data obtained under laboratory conditions (Fig. 3, Curve a) or under industrial conditions (Fig. 3, Curves b and c). Based on Eq. (15), the dependence of the optimum velocity on the diameter of the filter element and on the size of the particles to be removed is demonstrated in Fig. 4. The figure suggests that the velocity should decrease as the size of particles decreases in order to maintain the performance. Similarly, the dependence of the velocity on the strength of the magnetic field is represented in Fig. 5. It may be seen from the figure that the velocity can be increased as the strength of magnetic field increases. However, there seems to be no use in increasing the strength of the field above 0.9×10^5 A/m. Badescu and Rezlescu (13) showed experimentally that filter performance increases with the strength of the magnetic field up to 0.6×10^5 A/m. It should be noted here that the filtration velocities in Figs. 4 and 5 are the velocities which satisfy a filter performance of 80% or higher under the indicated conditions. These results are also in good agreement with experimental data given in the literature (9).

The effects of system parameters on the filtration velocity are investigated in this paper for the velocities up to 0.2 m/s, although in practical applications the filtration velocities are around 0.05–0.1 m/s. Generally, very good agreement with experimental data was observed, but small deviations were observed in some cases. If it is assumed that the experimental error is minor,

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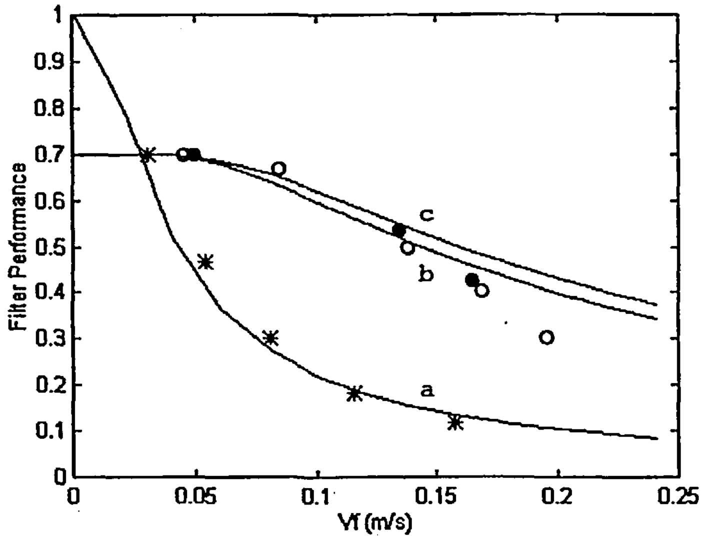

FIG. 3 Comparison of experimental data with the model predictions (from Eq. 17) for the dependence of the filter performance on the filtration velocity. (——) Model predictions. (*, O, ●) Experimental data (9). Curve a: Filtration of a suspension prepared from magnetite (Fe_3O_4) particles; $H = 30 \times 10^3$ A/m, $\chi = 0.4$, $d = 5.7 \times 10^{-3}$ m, $\rho_f = 1000$ kg/m^3, $\delta = 4 \times 10^{-6}$ m, $L = 0.042$ m, $\Phi = 1$. Curve b: Filtration of condensed water from a power plant; $H = 40 \times 10^3$ A/m ($\mu = 52$), $\chi = 0.17$, $d = 5 \times 10^{-3}$ m, $\rho_f = 1000$ kg/m^3, $\delta = 1 \times 10^{-6}$ m, $L = 1.0$ m, $\Phi = 0.7$. Curve c: Filtration of liquid ammoniac; $H = 80 \times 10^3$ A/m, $\chi = 0.1$, $d = 2.4 \times 10^{-3}$ m, $\rho_f = 590$ kg/m^3, $\delta = 1 \times 10^{-6}$ m, $L = 0.2$ m, $\Phi = 0.7$.

the differences between the experimental and the calculated velocities may originate from calculation of the highest velocity (V_0) used in the model. It was assumed in the model that the interstitial velocity is constant for a constant filtration velocity. In fact, the real porosity of a filter may decreases over time due to particle accumulation in the filter, and as a result the interstitial velocity (or the value of K_v) may increase over time. Therefore, the constant value of K_v may be the main source of error in the model calculations. However, the results discussed above confirm that the model is very convenient for estimating the filtration velocities under various filtration conditions.

The performance of a magnetic filter may be considered as the sum of the particle capture capabilities (elementary performance) of each filter element. This performance, and therefore the overall performance, decreases proportionally at velocities higher than those calculated from the equations given above. Without decreasing the filtration velocity, the overall performance

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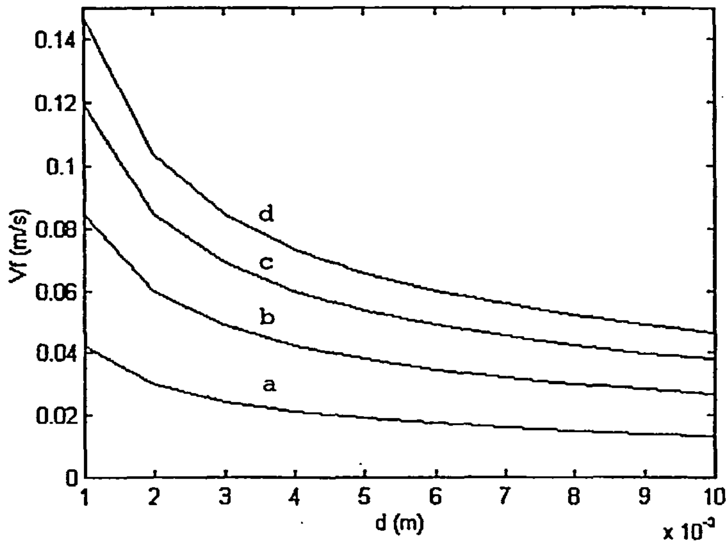

FIG. 4 The dependence of filtration velocity on the diameter of the filter element in the filtration of suspensions prepared with magnetite particles of different sizes. Curve a: $\delta = 1 \times 10^{-6}$ m, Curve b: $\delta = 4 \times 10^{-6}$ m, Curve c: $\delta = 8 \times 10^{-6}$ m, Curve d: $\delta = 12 \times 10^{-6}$ m. The other conditions are the same as in Fig. 3, Curve a.

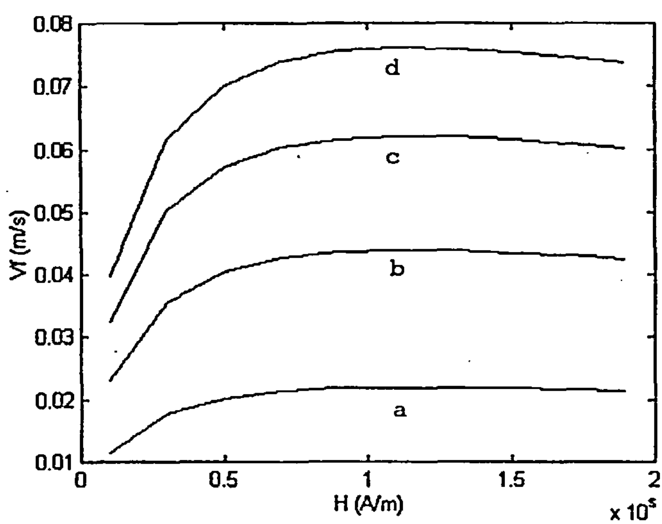

FIG. 5 The dependence of filtration velocity on the strength of the applied magnetic field in the filtration of suspensions prepared with magnetite particles of different sizes. The particle sizes are the same as in Fig. 4 and the other conditions are the same as in Fig. 3, Curve a.

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may be increased by increasing the number of the accumulation points, i.e., by increasing the active filter length. The flow velocity is not very dependent on the length (proportional to $L^{1/3}$) since the area normal to flow does not change much with increasing filter length.

CONCLUSION

A theoretical model has been developed for calculating the optimum filtration velocity in HGMF by using the boundary layer regime approach. The model is valid for cleaning fluids containing magnetic (ferro, para, or diamagnetic) particles through magnetic filters with packing fractions up to 0.62. It can be used to predict the optimum flow velocity in terms of magnetic, geometric, and rheological parameters of the filtration system. Theoretical and experimental results suggest that the optimum filtration velocity is proportional with $H^{0.7-1}$, $(1/d)^{2.7}$, $\delta^{0.5-0.7}$, and $L^{1/3}$. The optimum fluid velocity for a HGMF system can easily be estimated by means of the above equations if the values of the parameters of a system are known. These equations can be applicable for both short (model) and long (industrial) filters. The convenience of the theoretical approaches for calculating the fluid velocity has been confirmed by using experimental data given in the literature.

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NOTATION

a	radius of the filter elements (m)
C_0	total concentration of particles before filtration (g/kg)
C'_0	total concentration of magnetic particles before filtration (g/kg)
ΔC	decrease in the concentration after filtration
C	a constant with a value of 1.701 in Eq. (4)
d	diameter of the filter element (m), $d = 2a$
F_D	drag force (N)
F_{Dx}	drag component of the drag force (N)
F_{Dy}	lift component of the drag force (N)
F_G	gravitational force (N)
F_m	magnetic force (N)
g	gravity of acceleration (m/s^2)
HGMF	high gradient magnetic filtration
H	external magnetic field (A/m)
$\mathbf{H}$	dimensionless magnetic field strength
K_v	a constant for estimating the velocity profile within a porous filter
L	filter length in flow direction (m)

L_a	active filter length in flow direction (m)
L_1, L_2, L_3, L_4	characteristic distances as defined in Fig. 2 (m)
M	a correlation coefficient regarding the fluctuations in velocity
R	radial coordinate from the contacting point of the filter elements (m)
R_0	distance from the furthest accumulation layer to the contacting point of the filter elements (m)
Re_x	Reynolds number
X	distance from the leading edge of the accumulation layer (m)
V_f	filtration velocity (bulk flow velocity) (m/s)
V_i	average interstitial fluid velocity (m/s)
V_0	maximum interstitial velocity (m/s)
V_R	local interstitial fluid velocity at a distance of R (m/s)

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Greek Letters

δ	effective particle size (m)
ϵ	filter porosity
λ_x	a coefficient for drag components of the drag force
λ_y	a coefficient for lift components of the drag force
ρ_f, ρ_p	fluid density and particle density, respectively (kg/m^3)
μ	magnetic permeability of the filter elements
μ_0	the free space magnetic permeability $= 4\pi \times 10^{-7}$ (H/m)
η	viscosity of the fluid (N·s/m^2)
χ_p	particle magnetic susceptibility
χ_f	fluid magnetic susceptibility
χ	effective magnetic susceptibility ($\chi = \chi_p - \chi_f$)
τ_w	shear stress on the accumulation surface (N/m^2)
ψ	filter performance ($\psi = \Delta C/C_0$)
Φ	fraction of magnetic particles in the fluid ($\Phi = C_0'/C_0$)

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Received by editor May 25, 1997
Revision received September 1997

Downloaded At: 11:24 25 January 2011